A short note: For Question 1

Both Method are good with Consistency.

The FSCT has truncation error O(ht, hx2), so when ht→0, hx→0, error goes to 0.

The C-N has truncation error O(ht2, hx2), so when ht→0, hx→0, error goes to 0.

Problem 2 Sum\_up Sheng Xu

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|  | Discontinuous | C0 | C∞ |
| FSCT | Unstable  Unstable First Forward Center Discontinuity | Unstable  Unstable First Forward Center C0 | Unstable  Unstable First Forward Center C infinity |
| 1st Upwind CFL | Stable with dissipation  Stable First Order Upwind CFL + discontinuity | Stable with dissipation  Stable First Order Upwind CFL + C 1 | Stable  Stable First Order Upwind CFL + C infinity |
| 1st Upwind non-CFL | Unstable  Unstable First Order Upwind no CFL+discontinuous | Unstable  Unstable First Order Upwind no CFL+C1 | Unstable  Unstable First Order Upwind no CFL+C infinity2 |
| Crank-Nicholson | Stable with Dispersion  Stable C-N Dispersion discontinuos | Stable with dissipation  Stable C-N Dispersion C0 | Stable  Stable C-N C-infinity |
| BSCT | Stable with Dispersion  Stable Backward Center Dispersion discontinuos | Stable with Dispersion  Stable Backward Center Dispersion C1 | Stable  Stable Backward Center C infinity |

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| --- | --- | --- | --- | --- | --- |
| Functions | Discontinuous | | C0 | | C∞ |
| F=0  F=0.5 | when -1≤ x <-0.5  0≤ x < 0.5  when -0.5≤ x < 0  0.5≤x<1 | F=2x+2  F=-2x  F=2x  F=2-2x | when x≤ -0.5  when -0.5≤x<0  when 0≤x<0.5  when 0.5≤ x≤1 | cos(3x) |

Conclusion:

All stable methods exhibit certain dissipation with C∞ functions. FSCT and First Order Upwind method without CFL are never stable.

For Discontinuity and C1 functions, both implicit methods(C-N and BSCT) exhibits dispersion, making pictures less smooth. BSCT is more effected in C1 function, while C-N is more effected in Discontinuity function.